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HIGHER TWIST OPERATOR EFFECTS TO PARTON DENSITIES AT SMALL X

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We investigate the Q^2 evolution of parton distributions at small x values, obtained in the case of soft initial conditions. The contributions of twist-two and (renormalon-type) higher-twist operators of the Wilson operator product expansion are taken into account. The results are in very good agreement with deep inelastic scattering experimental data from HERA.

The measurements of the deep-inelastic scattering structure function (SF) F_2 in HERA [1] have permitted the access to a very interesting kinematical range for testing the theoretical ideas on the behavior of quarks and gluons carrying a very low fraction of momentum of the proton, the so-called small x region. The reasonable agreement between HERA data and the next-to-leading order (NLO) approximation of perturbative QCD that has been observed for $Q^2 \geq 2\text{GeV}^2$ (see [2] and references therein) indicates that perturbative QCD could describe the SF evolution up to very low Q^2 values.

The standard program [3,4] to study the behavior of quarks and gluons is carried out by comparison of data with the numerical solution of the DGLAP equations by fitting the parameters of the x profile of partons at some initial Q_0^2. However, if one is interested in analyzing exclusively the small x region ($x \leq 0.01$), there is the alternative of doing a simpler analysis by using some of the existing analytical solutions of DGLAP in the small x limit (see, for example,[5,2]). The main ingredients of the study [2] are:

- Both, the gluon and quark singlet densities are presented in terms of two components ($'+'$ and $'-'$) which are obtained from the analytical Q^2 dependent expressions of the corresponding ($'+'$ and $'-'$) parton distributions moments.
- The $'-'$ component is constant at small x, whereas the $'+'$ component grows at $Q^2 \geq Q_0^2$ as

$$\sim \exp\left(2\sqrt{\left[a_+ \ln\left(\frac{a_s(Q_0^2)}{a_s(Q^2)}\right) - \left(b_+ + a_+\frac{\beta_1}{\beta_0}\right)\left(a_s(Q_0^2) - a_s(Q^2)\right)\right]\ln\left(\frac{1}{x}\right)}\right),$$

where the LO term $a_+ = 12/\beta_0$ and the NLO one $b_+ = 412f/(27\beta_0)$. Here the coupling constant $a_s = \alpha_s/(4\pi)$, β_0 and β_1 are the first two coefficients of QCD β-function and f is the number of active flavors.

We shortly compile below the main results of [2] at the leading order (LO) approximation and demonstrate some new (preliminary) results, where the contributions of higher-twist (HT) operators (i.e. twist-four ones and twist-six ones) of the Wilson operator product expansion are taken into account in the framework of renormalon model (see [6]). The importance of the HT contributions at small-x has been demonstrated in [7].

1. Our purpose is to show the small x asymptotic form of parton distributions in the framework of the DGLAP equation starting at some Q_0^2 with the flat function:

$$f_a^{\tau 2}(Q_0^2) \;=\; A_a \qquad (\text{ hereafter } a=q,g), \tag{1}$$

where $f_a^{\tau 2}$ are the leading-twist (LT) parts of parton (quark and gluon) distributions (PD) multiplied by x and A_a are unknown parameters that have to be determined from data. Through this work at small x we neglect the non-singlet quark component [a].

The full small x asymptotic results for PD and SF F_2 at LO is:

$$F_2(x,Q^2) \;=\; e\cdot f_q(x,Q^2), \qquad f_a(x,Q^2) \;=\; f_a^{+}(x,Q^2) + f_a^{-}(x,Q^2)\,, \tag{2}$$

where the $'+'$ and $'-'$ components $f_a^{\pm}(x,Q^2)$ are given by the sum

$$f_a^{\pm}(x,Q^2) \;=\; f_a^{\tau 2,\pm}(x,Q^2) + f_a^{h\tau,\pm}(x,Q^2) \tag{3}$$

of the LT parts $f_a^{\tau 2,\pm}(x,Q^2)$ and the HT parts $f_a^{h\tau,\pm}(x,Q^2)$, respectively.

The small x asymptotic results for PD, $f_a^{\tau 2,\pm}$

$$f_g^{\tau 2,+}(x,Q^2) = \left(A_g + \frac{4}{9}A_q\right)\tilde{I}_0(\sigma)\; e^{-\overline{d}_+(1)s} \;+\; O(\rho) \quad , \tag{4}$$

$$f_q^{\tau 2,+}(x,Q^2) = \frac{f}{9}\left(A_g + \frac{4}{9}A_q\right)\rho\;\tilde{I}_1(\sigma)\; e^{-\overline{d}_+(1)s} \;+\; O(\rho)\,, \tag{5}$$

$$f_q^{\tau 2,-}(x,Q^2) = A_q e^{-d_-(1)s} \;+\; O(x),\;\; f_g^{\tau 2,-}(x,Q^2) \;=\; -\frac{4}{9}f_q^{\tau 2,-}(x,Q^2)\,, \tag{6}$$

where $\overline{d}_+(1) = 1 + 20f/(27\beta_0)$ and $d_-(1) = 16f/(27\beta_0)$ are the regular parts of d_+ and d_- anomalous dimensions, respectively, in the limit $n \to 1$ [b]. The

[a] We would like to note that new HERA data [1] show a rise of F_2 structure function at low Q^2 values ($Q^2 \sim 1\text{GeV}^2$) when $x \to 0$ (see Fig.1, for example). The rise can be explained in a natural way by incorporation of HT terms in our analysis (see Eqs.(7)-(9)).

[b] From now on, for a quantity $k(n)$ we use the notation $\hat{k}(n)$ for the singular part when $n \to 1$ and $\overline{k}(n)$ for the corresponding regular part.

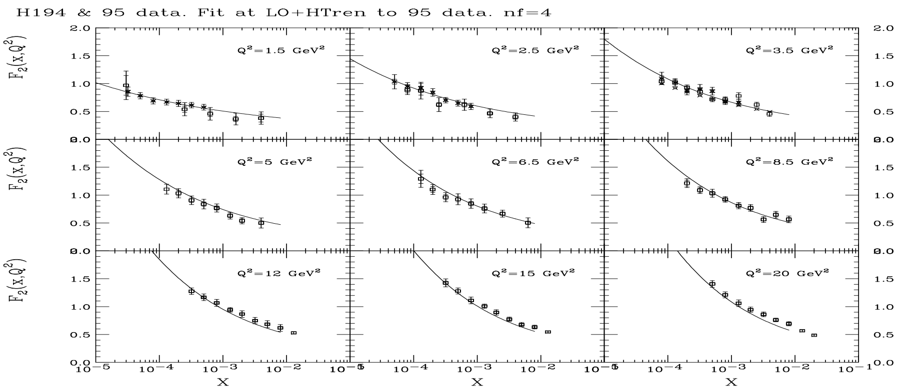

Figure 1: The structure function F_2 as a function of x for different Q^2 bins. The experimental points are from H1. The inner error bars are statistic while the outer bars represent statistic and systimatic errors added in quadrature. The curves are obtained from fits at LO when the HT contributions have been incorporated.

function $\tilde{I}_\nu$ ($\nu = 0, 1$) coincides with the modified Bessel function I_ν at $s \geq 0$ and the Bessel function J_ν at $s < 0$. Using the calculations [8,9], we show the HT effect in the renormalon case. We present the results only for the terms proportional of some power of $\ln(1/x)$ (full expressions can be found in the last papaer of [2]), making the following subtitutions in the corresponding LT results presented in Eqs.(4)-(6):

$f_a^{\tau 2,+}(x, Q^2)$ (see Eqs.(4),(5)) $\rightarrow f_a^{h\tau,+}(x, Q^2)$ by

$$A_a \left\{ \tilde{I}_0(\sigma),\ \rho \tilde{I}_1(\sigma) \right\} \rightarrow A_a \left\{ \frac{32f}{15\beta_0^2},\ \frac{256f}{45\beta_0^2} \right\} \left(\frac{\Lambda_{1,a}^2}{Q^2} - \frac{8}{7}\frac{\Lambda_{2,a}^4}{Q^4} \right) \frac{1}{\rho} \tilde{I}_1(\sigma) + ..., \quad (7)$$

where $\Lambda_{1,a}^2$ and $\Lambda_{2,a}^4$ are magnitudes of twist-four and twist-six corrections.

$f_g^{\tau 2,-}(x, Q^2)$ (see Eq.(6)) $\rightarrow f_g^{h\tau,-}(x, Q^2)$ by

$$A_q \ \rightarrow\ A_q \cdot \frac{32f}{15\beta_0^2} \left(\frac{\Lambda_{1,q}^2}{Q^2} - \frac{8}{7}\frac{\Lambda_{2,q}^4}{Q^4} \right) \ln\left(\frac{1}{x}\right) + \ ... \ . \quad (8)$$

$f_q^{\tau 2,-}(x, Q^2)$ (see Eq.(6)) $\rightarrow f_q^{h\tau,-}(x, Q^2)$ by

$$A_q \rightarrow \frac{128f}{45\beta_0^2} \left\{ A_q \cdot \left(\frac{\Lambda_{1,q}^2}{Q^2} \left[\ln\left(\frac{Q^2}{x\Lambda_{1,q}^2}\right) - \frac{209}{60} - \frac{8f}{81} \right] - \frac{8}{7}\frac{\Lambda_{2,q}^4}{Q^4} \left[\ln\left(\frac{Q^2}{x\Lambda_{2,q}^2}\right) - \frac{6517}{3150} - \frac{8f}{81} \right] \right) - \frac{2f}{9} A_g \cdot \left(\frac{\Lambda_{1,g}^2}{Q^2} - \frac{8}{7}\frac{\Lambda_{2,g}^4}{Q^4} \right) \right\} \ln\left(\frac{1}{x}\right) + \ ... \ . \quad (9)$$

2. With the help of the above equations we have analyzed F_2 HERA data at small x from the H1 collaboration. We have fixed the number of active flavors f=4 and $\Lambda_{\overline{\rm MS}}(n_f = 4) = 250$ MeV, which is a reasonable value extracted

from the traditional (higher x) experiments. Moreover, we put $\Lambda_{1,a} = \Lambda_{2,a}$ in agreement with [10].

The results are shown on Fig. 1. We found very good agreement between our approach based on QCD and HERA data. The (renormalon-type) HT terms lead to the natural explanation of the rise of F_2 structure function at low values of Q^2 and x.

As a next step of our investigations, we plan to finish this study and to investigate HT contributions to PD and SF relations, observed in [11,12].

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References

1. H1 Coll.: S. Aid *et al.*, *Nucl. Phys.* B **497**, 3 (1997); ZEUS Coll.: M. Derrick *et al.*, *Zeit. Phys.* C **7**, 609 (1999).
2. A.V. Kotikov and G. Parente, *Nucl. Phys.* B **549**, 242 (1999); hep-ph/9810223; *Nucl.Phys.Proc.Suppl.* B **99**, 146 (2000); hep-ph/0006197; hep-ph/0012299.
3. A.D. Martin *et al.*, *Eur.Phys.J.* C **14**, 133 (2000); M. Gluck *et al.*, *Eur. Phys. J.* C **5**, 461 (1998); H.L. Lai *et al.*, *Eur. Phys. J.* C **12**, 375 (2000).
4. G. Parente *et al.*, *Phys. Lett.* B **333**, 190 (1994); A.L. Kataev *et al.*, *Phys. Lett.* B **388**, 179 (1996); **417**, 374 (1998); *Nucl. Phys. Proc. Suppl.* **64**, 138 (1998); *Nucl. Phys.* B **573**, 405 (2000).
5. A. De Rújula *et al.*, *Phys. Rev.* D **10**, 1649 (1974); R.D. Ball and S. Forte, *Phys.Lett.* B **336**, 77 (1994); L. Mankiewicz et al., *Phys.Lett.* B **393**, 175 (1997).
6. M. Beneke, *Phys. Report* **317**, 1 (1991); M. Beneke and V.M. Braun, hep-ph/0010288.
7. J. Bartels, *Phys.Lett.* B **298**, 204 (1993); *Zeit. Phys.* C **60**, 471 (1993); E.M. Levin *et al.*, *Nucl. Phys.* B **387**, 589 (1992).
8. E. Stein *et al.*, *Nucl.Phys.* **B536**, 318 (1998).
9. A.V. Kotikov, *Phys.Atom.Nucl.* **57**, 133 (1994); *Phys.Rev.* D **49**, 5746 (1994).
10. M. Dasgupta and B.R. Webber, *Phys. Lett.* B **382**, 273 (1996).
11. A.V. Kotikov, *JETP Lett.* **59**, 667 (1994); A.V. Kotikov and G. Parente, *Phys. Lett.* B **B379**, 195 (1996).
12. A.V. Kotikov, *JETP* **80**, 979 (1995); A.V. Kotikov and G. Parente, hep-ph/9608409; *Mod. Phys. Lett.* A **12**, 963 (1997); *JETP* **85**, 17 (1997); hep-ph/9609439.